Name solutions

EE 311 | Final Exam | Spring 2012

May 1, 2012

Closed Text and Notes, No calculators

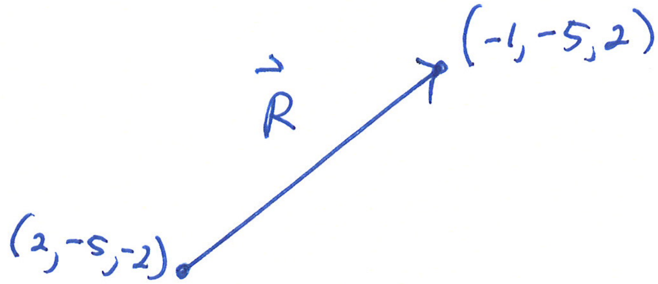
1) Be sure you have 15 pages and the additional pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

5) This exam is worth 150 points.

(5 pts) 1. Find the unit vector directed from the point (2, -5, -2) toward the point (-1, -5, 2) in rectangular coordinates.



$$\vec{R} = (-1-2)\hat{a}_x + (-5-(-5))\hat{a}_y + (2-(-2))\hat{a}_z$$

$$= -3\hat{a}_x + 4\hat{a}_z$$

$$R^2 = \vec{R}\cdot\vec{R} = 9+16=25$$

so $R = 5$

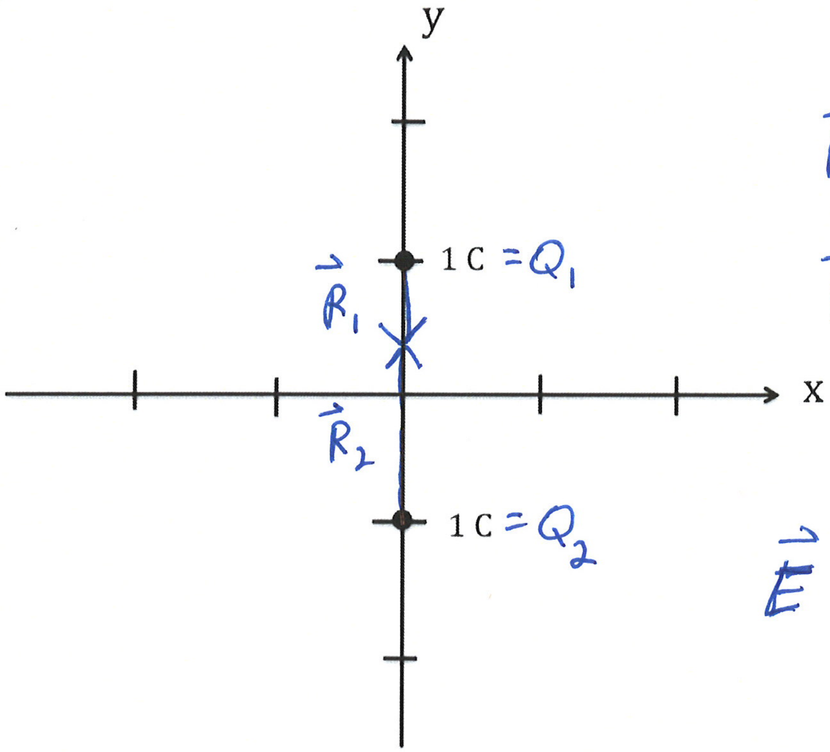
$$\hat{a}_R = \frac{\vec{R}}{R} = \frac{-3\hat{a}_x + 4\hat{a}_z}{5}$$

$$= -\frac{3}{5}\hat{a}_x + \frac{4}{5}\hat{a}_z$$

(5 pts) 2. What is the intersection of the surfaces r = 1m and $\theta = \frac{\pi}{3}$?

circle

(12 pts) 3. There is a charge of 1 C at (0, 1m, 0) and at (0, -1m, 0) as shown. Find $\mathbf{E}(0,y,0)$, the electric field intensity on the y-axis, for -1m < y < 1 m.



$$\vec{R}_1 = (y-1)\hat{a}_y$$

$$R_1 = 1-y$$

$$\vec{R}_2 = (y+1)\hat{a}_y$$

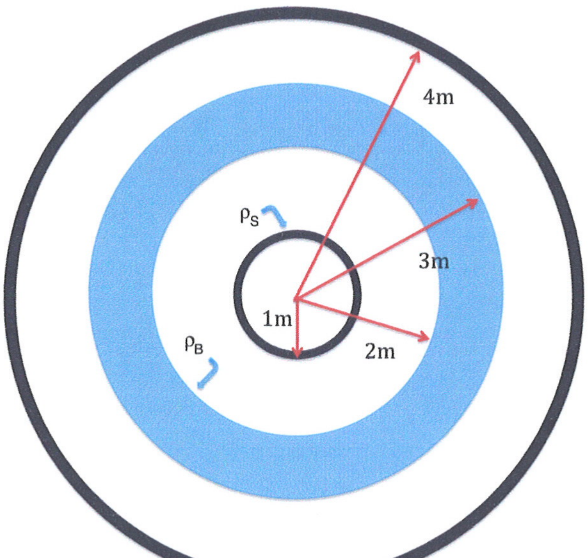
$$R_2 = y+1$$

$$\vec{E} = \sum_{k=1}^{2} \frac{Q_k}{4\pi\epsilon_0 R_k^3}\vec{R}_k$$

$$E = \frac{1\,C}{4\pi\epsilon_0(1-y)^3}(y-1)\hat{a}_y + \frac{1\,C}{4\pi\epsilon_0(y+1)^3}(y+1)\hat{a}_y$$

$$= \left[\frac{-1\,C}{4\pi\epsilon_0(1-y)^2} + \frac{1\,C}{4\pi\epsilon_0(1+y)^2}\right]\hat{a}_y \quad -1m < y < 1m$$

(10 pts) 4. A capacitor consists of two co-centric spheres as shown. The outer radius of the inner conductor is $r_A = 1\text{m}$, the inner radius of the outer conductor is 4m, and there are two air gaps and a dielectric from 2m < r < 3m. A voltage is applied such that a surface charge of $\rho_S = (\frac{2\text{x}10^{-9}}{\pi})\frac{\text{C}}{\text{m}^2}$ appears on the outer surface of the inner conductor and a bound charge of $\rho_B = -(\frac{10^{-9}}{4\pi})\frac{\text{C}}{\text{m}^2}$ on the inside surface, $r_B = 2\text{m}$, of the dielectric. Find the electric field intensity, the electric flux density and the polarization for 1m < r < 4m. Note $\varepsilon_o = \frac{10^{-9}}{36\pi}\frac{\text{F}}{\text{m}}$.



$$Q_s = \rho_s 4\pi r_A^2 = \frac{2\times10^{-9}}{\pi}\frac{c}{m^2}4\pi(1m)^2$$

$$Q_s = 8\times10^{-9}\,c$$

$$\vec{D} = \frac{8\times10^{-9}c}{4\pi r^2}\hat{a}_r \quad \text{for } 1m < r < 4m$$

$$Q_B = \rho_B 4\pi r_B^2 = -\left(\frac{10^{-9}}{4\pi}\frac{c}{m^2}\right)4\pi(2m)^2$$

$$= -4\times10^{-9}\,c$$

$$P_B = 0 \quad \text{for } 1m < r < 2m \text{ and } 3m < r < 4m$$

$$\frac{4\times10^{-9}c}{4\pi r^2}\hat{a}_r \quad \text{for } 2m < r < 3m$$

$$E = \frac{D-P}{\epsilon_o}$$

$$\vec{E} = \frac{1}{\epsilon_o}\left(\frac{8\times10^{-9}c}{4\pi r^2} - \frac{4\times10^{-9}c}{4\pi r^2}\right)\hat{a}_r \quad 2m < r < 3m$$

$$\vec{E} = \left(\frac{36\pi}{10^{-9}}\frac{m}{F}\right)\left(\frac{4\times10^{-9}c}{4\pi r^2}\right)\hat{a}_r = \frac{36}{r^2}\hat{a}_r\frac{V}{m}$$

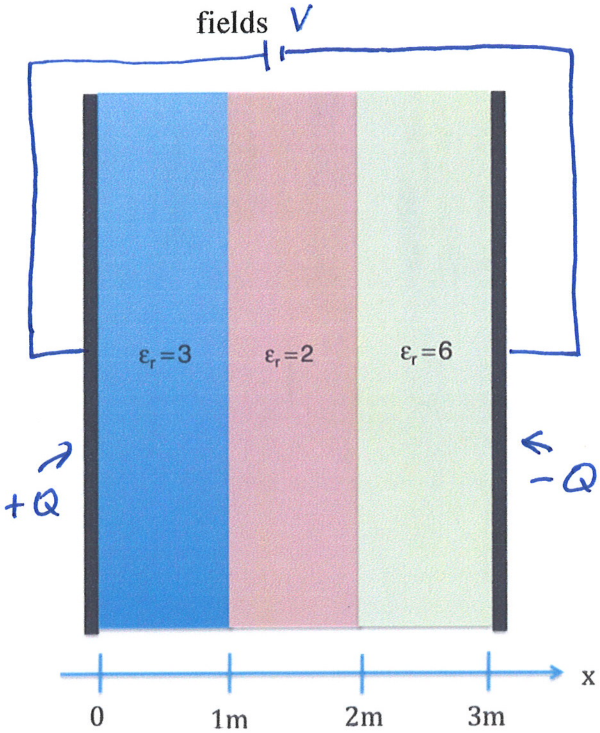
for the gaps $\vec{E} = \frac{\vec{D}}{\epsilon_o} = \frac{36\pi}{10^{-9}}\frac{8\times10^{-9}}{4\pi r^2}\hat{a}_r = \frac{72}{r^2}\hat{a}_r\frac{V}{m}$

so,

$$\vec{E} = \frac{72}{r^2}\hat{a}_r\frac{V}{m} \quad \text{for } 1m < r < 2m \text{ and } 3m < r < 4m$$

$$= \frac{36}{r^2}\hat{a}_r\frac{V}{m} \quad \text{for } 2m < r < 3m$$

(10 pts) 5. Using field concepts, find the value of the capacitance of the structure shown below. The plates are parallel and of area $36\pi\,\text{m}^2$, Between the plates the value of the relative dielectric constants are $\varepsilon_r = 3$ for $0 < x < 1\text{m}$, $\varepsilon_r = 2$ for $1\text{m} < x < 2\text{m}$, and $\varepsilon_r = 6$ for $2\text{m} < x < 3\text{m}$. Ignore fringing fields



$$\rho_s(0) = \frac{Q}{36\pi m^2}$$

$$\vec{D} = \frac{Q}{36\pi m^2}\hat{a}_x \quad 0 < x < 3m$$

$$\vec{E} = \frac{Q}{3\epsilon_0 36\pi m^2}\hat{a}_x \quad 0 < x < 1m$$

$$= \frac{Q}{2\epsilon_0 36\pi m^2}\hat{a}_x \quad 1m < x < 2m$$

$$= \frac{Q}{6\epsilon_0 36\pi m^2}\hat{a}_x \quad 2m < x < 3m$$

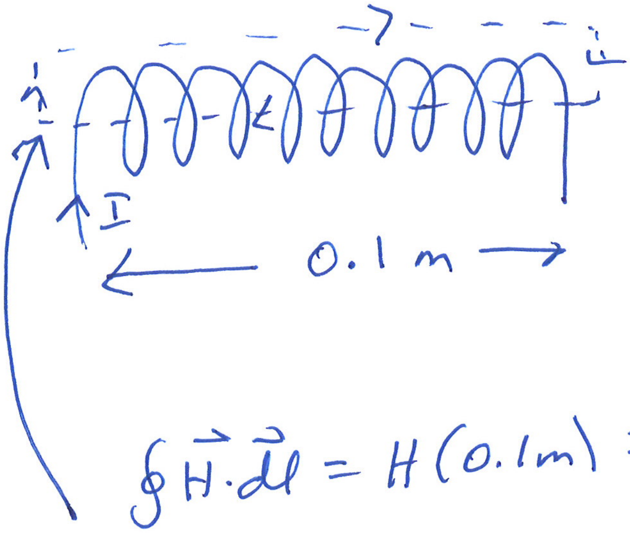
$$V = -\int_{3m}^{1m} \vec{E}\cdot\vec{dx} = -\int_{3m}^{2m} \frac{Q\,dx}{6\epsilon_0 36\pi m^2} - \int_{2m}^{1m} \frac{Q\,dx}{2\epsilon_0 36\pi m^2} - \int_{1m}^{0} \frac{Q\,dx}{3\epsilon_0 36\pi m^2}$$

$$= \frac{Q}{6\epsilon_0 36\pi m} + \frac{Q}{2\epsilon_0 36\pi m} + \frac{Q}{3\epsilon_0 36\pi m} = \frac{6Q}{6\epsilon_0 36\pi m}$$

$$C = \frac{Q}{V} = 36\pi\epsilon_0 m = 36\pi\frac{10^{-9}}{36\pi}\frac{F}{m} m$$

$$C = 10^{-9} F$$

(10 pts) 6. A tightly wound solenoid consists of 1000 turns, is 0.1 m long and has a radius of 0.01m. Around this solenoid is another tightly wound solenoid of 400 turns, length of 0.04m and radius 0.02m. What is the mutual inductance of these solenoids?



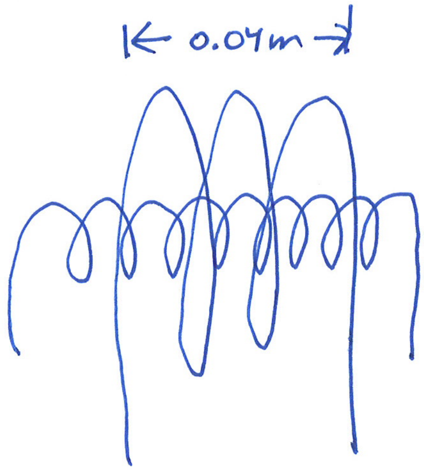
$\leftarrow \text{Area} = \pi r^2 = \pi (0.01m)^2 = 10^{-4}\pi \, m^2$

$\oint \vec{H}\cdot d\vec{l} = H(0.1m) = I_{encl.} = 1000 I$

$H = \frac{1000 I}{0.1m} = 10^4 I$

$B = \mu_0 H = \mu_0 10^4 I$

$\Psi = B(\text{Area}) = (\mu_0 10^4 I)(10^{-4}\pi) = \mu_0 \pi I$



$\lambda = N\Psi = 400 \mu_0 \pi I$

$M = \frac{\lambda}{I} = \frac{400 \mu_0 \pi I}{I}$

$M = 400 \mu_0 \pi$

(6 pts) 7. Fill in the table with the standard units for the following

magnetic flux density, **B**	$T = \frac{Wb}{m^2}$
Magnetic field intensity, **H**	$\frac{A}{m}$
Electric Field Intensity, **E**	$\frac{V}{m}$
Electric Flux Density, **D**	$\frac{C}{m^2}$
Electric flux, **Ψ**	C
Magnetic flux, **Ψ**	Wb

(5 pts) 8. For the region of space r < 1m, the total charge in this region is decreasing at the rate of $4\pi \frac{\mu C}{s}$.

What is the current density on the surface r = 1m?

If Q = charge inside region r < 1m

$\frac{dQ}{dt} = 4\pi \frac{\mu C}{s}$ = current flowing in $\hat{a}_r$ direction through the surface r = 1m

$$J = \frac{\left(\frac{dQ}{dt}\right)}{4\pi (1m)^2} = \frac{4\pi \times 10^{-6} \frac{C}{s}}{4\pi \ m^2} = 10^{-6} \frac{A}{m^2}$$

$$\vec{J} = 10^{-6} \hat{a}_r \frac{A}{m^2}$$

(10 pts) 9. A sphere of radius 1m has a charge on it of $\frac{10^{-9}}{3}$ C. If $V(\infty) = 10\text{V}$, find V for $0<r<\infty$. Note $\varepsilon_o = \frac{10^{-9}}{36\pi}\frac{\text{F}}{\text{m}}$.

$$\vec{E} = 0 \quad \text{for} \quad r < 1\text{m}$$

$$= \frac{\frac{10^{-9}}{3}}{4\pi\varepsilon_o r^2} = \frac{10^{-9}}{12\pi\left(\frac{10^{-9}}{36\pi}\right)r^2} = \frac{3}{r^2} \quad \text{for } r > 1\text{m}$$

$$V(r) - V(\infty) = -\int_\infty^r \vec{E} \cdot \vec{dl}$$

for $r > 1\text{m}$

$$V(r) - V(10\text{m}) = -\int_\infty^r \frac{3}{r^2} dr = \frac{3}{r}\Big|_\infty^r = \frac{3}{r}$$

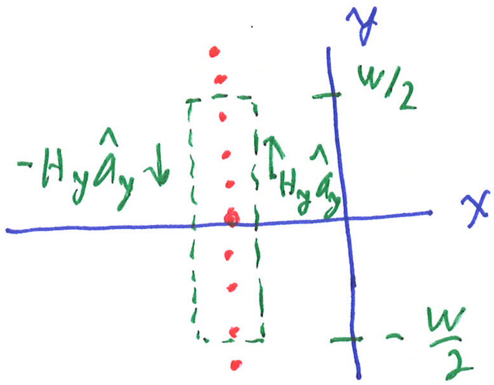
$$V(r) = \left[\frac{3}{r} + 10\right] \text{V} \quad \text{for} \quad r > 1\text{m}$$

for $r < 1\text{m}$ $\quad V(r) = V(1\text{m}) = \left[\frac{3}{1} + 10\right] \text{V}$

$$V(r) = 13\text{V} \quad \text{for} \quad r \leq 1\text{m}$$

(9 pts) 10. There is a sheet current density of $\mathbf{K} = 2\,\hat{\mathbf{a}}_z \frac{A}{m}$ flowing on the x = -1m and x = 1 m planes. Find the magnetic field intensity everywhere.

First find H due to the $x = -1m$ current plane



$$\oint \vec{H}\cdot\vec{dl} = H_y w + H_y w = 2\frac{A}{m} w$$

$$H_y = 1\frac{A}{m}$$

$$\vec{H} = 1\,\hat{a}_y \frac{A}{m} \qquad x > -1m$$

$$-1\,a_y \frac{A}{m} \qquad x < -1m$$

Similarly for the $x = 1\,m$ plane

$$\vec{H} = 1\,a_y \frac{A}{m} \qquad x > 1\,m$$

$$-1\,a_y \frac{A}{m} \qquad x < 1\,m$$

Using superposition, the H field everywhere due to the two current planes

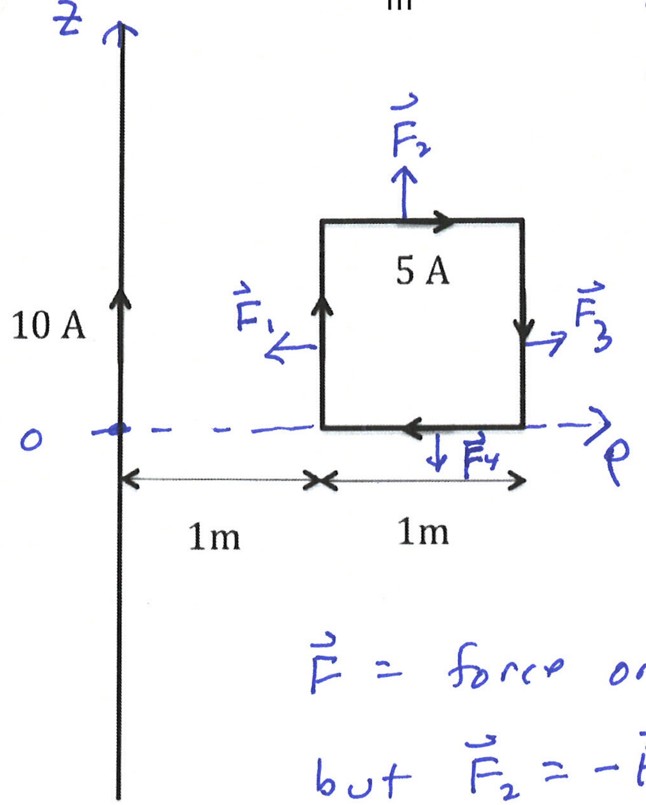
$$\vec{H} = 2\,\hat{a}_y \frac{A}{m} \qquad x > 1m$$

$$0 \qquad -1m < x < 1m$$

$$-2\hat{a}_y \frac{A}{m} \qquad x < -1m$$

(10 npts) 11. Find the force on the 1m x 1m loop shown. The loop and infinite wire are in the same plane.

Note $\mu_o = 4\pi x 10^{-7} \frac{H}{m}$



First find the magnetic flux density, $\vec{B}$, due to the 10A infinite wire.

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{10}{2\pi\rho}\hat{a}_\phi$$

$$\vec{B} = \mu_o \vec{H} = (4\pi \times 10^{-7})\frac{5}{\pi\rho}\hat{a}_\phi$$

$$= \frac{2\times 10^{-6}}{\rho}\hat{a}_\phi \ T$$

$\vec{F}$ = force on the loop $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

but $\vec{F}_2 = -\vec{F}_4$ so $\vec{F} = \vec{F}_1 + \vec{F}_3$

$$\vec{F}_1 = \int_0^{1m} I\vec{dl} \times \vec{B}$$

$$= \int_0^{1m} (5A)\, dz\, \hat{a}_z \times \frac{2\times 10^{-6}}{\rho} T\hat{a}_\phi$$

$$= \int_0^{1} (5A)\frac{2\times 10^{-6}}{1} T\, dz(-\hat{a}_\rho)$$

$$= -10^{-5}\hat{a}_\rho \ N$$

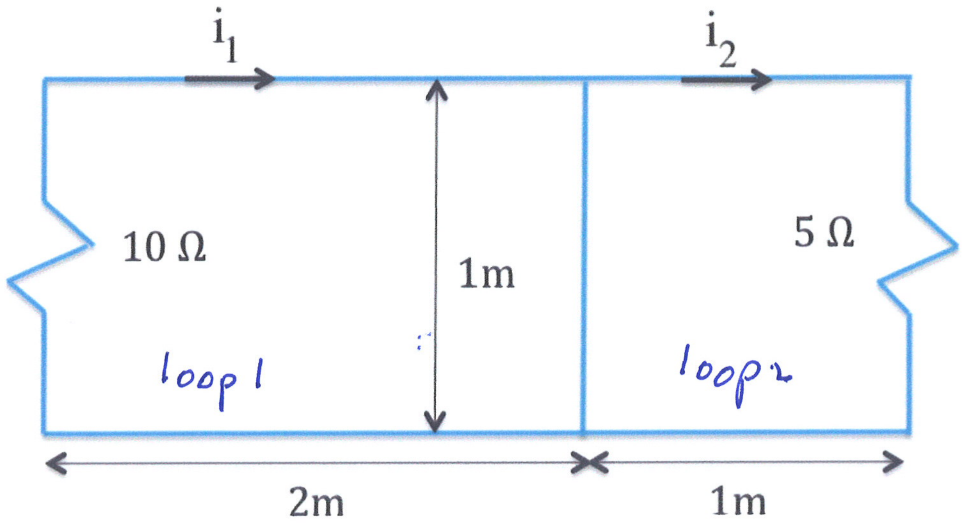
$$\vec{F}_3 = \int_{1m}^{0} (5A)\, dz\, \hat{a}_z \times \frac{2\times 10^{-6}}{\rho} T\hat{a}_\phi$$

$$= \int_{1m}^{0} (5A)\frac{2\times 10^{-6}}{2} T\, dz\, \hat{a}_\rho$$

$$= 5\times 10^{-6}\hat{a}_\rho \ N$$

$$F = \vec{F}_1 + \vec{F}_3 = -5\times 10^{-6}\hat{a}_\rho \ N$$

(12 pts) 12. The two-resistor circuit is in the field of magnitude $B = 20t \frac{Wb}{m^2}$ that is out of the page. Determine i_1 and i_2.



clockwise around loop 1

$$\oint \vec{E} \cdot \vec{dl} = -\left(\frac{d\varphi}{dt}_{\text{into page}}\right)$$

$$i_1(10\Omega) = -\left(-20\frac{V}{m^2}\right)(2m^2) = 40$$

$$i_1 = 4A$$

clockwise around loop 2

$$\oint E \cdot dl = -\left(\frac{d\varphi}{dt}_{\text{into page}}\right)$$

$$i_2(5\Omega) = -\left(-20\frac{V}{m^2}\right)(1m^2) = 20V$$

$$i_2 = 4A$$

(18 pts) 13. The magetic field intensity is $\mathbf{H}=10\frac{A}{m}\cos(\pi 10^4 t-\pi 10^{-4}x)\hat{\mathbf{a}}_z$ in a nonmagnetic dielectric.

Note $\frac{1}{\sqrt{\mu_o \varepsilon_0}} = 3\text{x}10^8 \frac{m}{s}$ and $\sqrt{\frac{\mu_o}{\varepsilon_o}} = 377\Omega$

(4 pts) A) What is the frequency in Hz?

$$\omega = 2\pi f = \pi 10^4$$

$$f = 5{,}000 \text{ Hz}$$

(4 pts) B) What is the wavelength?

$$\beta = \frac{2\pi}{\lambda} = \pi 10^{-4}$$

$$\lambda = 2\times 10^4 \text{ m}$$

(4 pts) C) What is the velocity?

$$u = \frac{\omega}{\beta} = \frac{\pi 10^4}{\pi 10^{-4}} = 10^8 \frac{m}{s}$$

$$\vec{u} = 10^8 \hat{a}_x \frac{m}{s}$$

(6 pts) D) Find the electric field intensity.

$$u = \frac{1}{\sqrt{\mu_o \epsilon}} = \frac{1}{\sqrt{\epsilon_r}\sqrt{\mu_o \epsilon_o}} = \frac{3\times 10^8 \frac{m}{s}}{\sqrt{\epsilon_r}} = 1\times 10^8 \frac{m}{s}$$

$$\epsilon_r = 9 \qquad \eta = \sqrt{\frac{\mu_o}{\epsilon}} = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_o}{\epsilon_o}} = \frac{377\Omega}{3} = 125.7\Omega$$

$$\vec{E} = +(10\tfrac{A}{m})(125.7\Omega)\cos(\pi 10^4 t - \pi 10^{-4} x)\hat{a}_y$$

$$\vec{E} = +1257\frac{V}{m}\cos(\pi 10^4 t - \pi 10^{-4} x)\hat{a}_y$$

(13 pts) 14. The z = 0 plane is the boundary between free space, z < 0, and a lossless, nonmagnetic dielectric for z > 0. The velocity of an EM wave in the dielectric is $1.5\text{x}10^8 \frac{\text{m}}{\text{s}}$.

Note $\frac{1}{\sqrt{\mu_o \varepsilon_0}} = 3\text{x}10^8 \frac{\text{m}}{\text{s}}$ and $\sqrt{\frac{\mu_o}{\varepsilon_o}} = 377\,\Omega$

(6 pts) A) For a uniform plane wave incident from the free space, at normal incidence, and with a wavelength of 5 m, what would be the wavelength of the transmitted wave?

in free space $f\lambda = f(5\text{m}) = c = 3\times10^8 \frac{m}{s}$

$f = 6\times10^7 \, s^{-1}$

in the dielectric $f\lambda = (6\times10^7 s^{-1})\lambda = 1.5\times10^8 \frac{m}{s}$

$\lambda = \frac{1.5}{6} \times 10 \, m = 2.5 \, m$

(7 pts) B) What is the reflection coefficient for a UPW incident from free space?

First find ϵ_{r_2} $\quad 1.5\times10^8 \frac{m}{s} = \frac{1}{\sqrt{\mu_o \epsilon_{r_2} \epsilon_o}} = \frac{3\times10^8 \frac{m}{s}}{\sqrt{\epsilon_{r_2}}}$

so $\epsilon_{r_2} = 4$

$\eta_2 = \sqrt{\frac{\mu_o}{\epsilon_{r_2}\epsilon_o}} = \frac{1}{2}(377\Omega) = \frac{1}{2}\eta_o$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{2}\eta_o - \eta_o}{\frac{1}{2}\eta_o + \eta_o} = \frac{-(\frac{1}{2})}{(\frac{3}{2})}$

$\Gamma = -\frac{1}{3}$

(10 pts) 15. The region of space for $z < 0$ is filled with a lossless nonmagnetic dielectric.

(5 pts) (A) If there is a perfect conductor at $z = 0$, what is the standing wave ratio for $z < 0$?

∞

(5 pts) (B) If the region for $z > 0$ is also a lossless nonmagnetic dielectric, that has the same dielectric constant as the dielectric in region $z < 0$, what is the standing wave ratio for $z < 0$?

1

(5 pts) 16. In a certain material the electric field is given by

$E(z,t) = 10e^{-0.01z}\cos[(2\times 10^8 s^{-1})t - (2m^{-1})z]\mathbf{a}_x \frac{V}{m}$. What is the skin depth?

$+0.01z = +0.01\delta = 1$

$\delta = \delta = \frac{1}{0.01} = 100\,m$